Smoothing methods of multivariate functions

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Let $\{\varphi_1, ...\varphi_m\}$ be a system of linear independent on the bounded set $G \subset \mathbf{R}_d$ functions, $\Phi_m = span\{\varphi_1, ...\varphi_m\}$, $\Phi_0 = \{0\}$, and F_{mp} - the set of convolution type functions

$$f(x) = f_0(x) + \sum_{i=1}^{k} \int_{G_i} f_i(t) K_i(x,t) d\mu_i \qquad \|f\|_p^* := \sum_{i=1}^{k} (\|f_i\|_{p_i})^{p_i} / p_i,$$

where $f_0 \in \Phi_m$, $f_i \in L_{p_i}(G_i, \mu_i)$, $K_i \in L_{p_i'}(G_i, \mu_i)$, G_i - bounded measurable set from $\mathbf{R}_{d_i}$, μ_i - positive measure, $1 < p_i < \infty$, $1/p_i + 1/p_i' = 1$.Let us given the information about elements of F_{mp} as the mapping $\mathcal{I} : F_{mp} \to \mathbf{R}_n$, such that the image $\mathcal{I}(f)$ is some ball of radius $\varepsilon(f) = (\varepsilon_1(f), ..., \varepsilon_n(f))$ and center $I(f) = (I_1(f), ..., I_n(f))$: $\mathcal{I}(f) = \{y \in \mathbf{R}_n : \ |y, I(f)|_{\varepsilon q}^* \le 1\}$, where

$$|x, y|_{\varepsilon q}^* = \sum_{i=1}^{l} \sum_{j=n_{i-1}+1}^{n_i} (|x_j - y_j| / \varepsilon_j)^{q_i} / q_i,$$

$1 < q_i < \infty$, $1/q_i + 1/q_i' = 1$, integers $n_i \ge 0$,($n_0 = 0$, $n_l = n$, $l \ge 1$) are fixed. I_j are linear functionals and $K_{ij}(t) = I_j(K_i(\cdot, t)) \in L_{p_i'}(G_i, \mu_i)$. Assume

$$span\{v_1, ..., v_m\} = m_0 \qquad (m_0 \le m), \quad v_i = (I_1(\varphi_i), ..., I_n(\varphi_i)).$$

From each system of functions $\{K_{i1}, ..., K_{in}\}$ extract linear independent on G_i subsystem

$$\{K_{ii_1}, ..., K_{ii_{r_i}}\} \qquad (i = 1 : k), \quad \sum_{i=1}^{k} r_i =: r_0.$$

and introduce the subset of functions from F_{mp}

$$S_{mp}^n = \{s \in F_{rp} \ : \ s_i(t) = |\sigma_i(t)|^{p_i'-1} sgn\, \sigma_i(t) \ \ (i = 1 : k)\}, \quad \sigma_i(t) = \sum_{j=1}^{r_i} c_j K_{ii_j}(t).$$

We find the criterion of solution of following smoothness problem:

$$\inf\, (\|f\|_p^* + |y, \mathcal{I}_n(f)|_{\varepsilon q}^* \ | \ f \in F_{mp}) =: N_{\varepsilon y}^{pq}.$$

It has a unique solution s_y, $N_{\varepsilon y}^{pq} = \|s_y\|_p^* + |y, \mathcal{I}_n(s_y)|_{\varepsilon q}^*$,where $s_y \in S_{mp}^n$ and satisfies the following equations:

$$I_j(s) + |c_j(s)|^{q_i'-1} sgn\, c_j(s) \varepsilon_j^{q_i'} = y_j, \quad j = n_{i-1} + 1 : n_i, \quad i = 1 : l,$$

$$\sum_{j=1}^{n} c_j(s) I_j(\varphi_\nu) = 0, \qquad \nu = 1 : m_0.$$

For $\varepsilon = 0$ the element $s_y \in S_{mp}^n$ interpolates given y: $I(s_y) = y$.