Kolmogorov diameters asymptotics for analytic and harmonic functions

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Let K be a compact set in $D \subset \mathbb{C}^n$ and A_K^D a compact subset in $C(K)$ consisted of all restrictions of functions, analytic in D with absolute value bounded by 1. Kolmogorov raised the problem about strong asymptotics for ε-entropy of this set: $H_\varepsilon\left(A_K^D\right) \sim \tau \left(\ln \frac{1}{\varepsilon}\right)^{n+1}$, and conjectured that, for $n = 1$, the constant τ coincides with the Green capacity $\tau(K, D)$ of the condenser (K, D). The above asymptotics is equivalent (Levin-Tikhomirov [1]) to the following asymptotics for Kolmogorov diameters (see,e.g. Tikhomirov [2]):

$$-\ln d_s\left(A_K^D\right) \sim \sigma\, s^{1/n},\ s \to \infty. \tag{1}$$

with $\sigma = \left(\frac{2}{(n+1)\tau}\right)^{1/n}$. Kolmogorov's hypothesis has been confirmed by efforts of many authors (Erokhin, Babenko, Zaharyuta, Levin-Tikhomirov, Widom, Nguyen, Skiba - Zahariuta, Fisher - Miccheli, et al).

In [6] (see, also [7]) it was conjectured that for a "good" pair $K \subset D$ in a Stein manifold Ω, $\dim \Omega = n$, the asymptotics (1) holds with $\sigma = 2\pi \left(\frac{n!}{C(K,D)}\right)^{1/n}$, where $C(K, D)$ is the pluricapacity of the "pluricondenser" (K, D), introduced by Bedford-Taylor; the problem has been reduced there to some problem of Pluripotential Theory (a multidimensional analogue of the one-dimensional fact about approximation of Green potential by harmonic functions with finite set of logarithmic poles). The last problem has been solved recently by Nivoche-Poletsky ([3, 4]), that gives, together with the results [6, 7], a final proof of our conjecture about strong asymptotics (1) under some natural restrictions about K, D. In our talk we are going to throw light upon above results and to discuss the problem about the asymptotics of Kolmogorov diameters of the similar compact Γ_K^D for harmonic functions in $\mathbb{R}^n$(in particular, the recent result [5]).

References

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