Convex and coconvex polynomial approximation

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We intend to present a survey of recent results on shape-preserving approximation by algebraic polynomials, obtained by Kopotun, Leviatan and the author. Say, let $\Delta^{(2)}$ be the set of convex functions f on $[-1,1]$, $\mathcal{P}_n$ be the space of all algebraic polynomials of degree $\le n$,

$$E_n(f) := \inf_{P_n \in \mathcal{P}_n} |f - P_n|_{C[-1,1]}, \quad E_n^{(2)}(f) := \inf_{P_n \in \mathcal{P}_n \cap \Delta^{(2)}} |f - P_n|_{C[-1,1]},$$

be the error of the best and the best convex approximation by algebraic polynomials respectively. Below let $f \in C^{(4)}(-1,1)$ be such that $|(1-x^2)f^{(4)}(x)| \le 1$, $x \in (-1,1)$. Then it is well-known, that

$$E_n(f) \le cn^{-4}, \quad n \ge N,$$

where c is an absolute constant, and $N = 3$. On the other hand, generally speaking, the same estimate is invalid if one replaces E_n by $E_n^{(2)}$, and N by any absolute constant. Anyway, the following is true: if $f \in \Delta^{(2)}$, then

$$E_n^{(2)}(f) \le cn^{-4}, \quad n \ge N(f),$$

where c is an absolute constant, and $N(f) = const$, depending on f.