Optimal approximation of elliptic problems by linear and nonlinear mappings

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(Joint work with S. Dahlke and W. Sickel)

We study the optimal approximation of the solution of an operator equation $\mathcal{A}(u) = f$ by linear mappings of rank n and compare this with the best n-term approximation with respect to an optimal Riesz basis. We consider worst case errors, where f is an element of the unit ball of a Hilbert space. We apply our results to boundary value problems for elliptic PDEs that are given by an isomorphism $\mathcal{A} : H_0^s(\Omega) \to H^{-s}(\Omega)$, where $s > 0$ and Ω is an arbitrary bounded Lipschitz domain in $\mathbb{R}^d$. Here we prove that approximation by linear mappings is as good as the best n-term approximation with respect to an optimal Riesz basis. We discuss why nonlinear approximation still might be very important for the approximation of elliptic problems. Our results are concerned with approximations and their errors, not with their numerical realization.