Non-linear approximation using sets of finite pseudo-dimension and Learning Theory

Maiorov V. (Haifa, Israel)

We consider optimal non-linear approximations of multivariate functions by manifolds of finite pseudo-dimension. For example, the spaces of univariate spline-functions with n free knots or rational functions of degree n are manifolds of pseudo-dimension of order n. In this paper we adduce some examples of manifolds in the multivariate case. We obtain asymptotic estimates for the pseudo-dimensional n-widths of Sobolev's classes $W_p^{r,d}$ in the space L_q in the case of non-compact embedding, that is when $r/d = 1/p - 1/q$. Note that in the case the well-known n-widths, including the Kolmogorov and entropy n-widths, do not converge to zero.

Also we consider the problem of learning neural networks from samples. Using a estimates of the psedo-dimension of the neural networks manifolds and approximation by the manifold, we estimate the sample size which is sufficient for obtaining the almost-optimal stochastic approximation of function classes. These results can be used to solve Smale's network problem.