

# Non-linear approximation using sets of finite pseudo-dimension and Learning Theory

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We consider optimal non-linear approximations of multivariate functions by manifolds of finite pseudo-dimension. For example, the spaces of univariate spline-functions with  $n$  free knots or rational functions of degree  $n$  are manifolds of pseudo-dimension of order  $n$ . In this paper we adduce some examples of manifolds in the multivariate case. We obtain asymptotic estimates for the pseudo-dimensional  $n$ -widths of Sobolev's classes  $W_p^{r,d}$  in the space  $L_q$  in the case of non-compact embedding, that is when  $r/d = 1/p - 1/q$ . Note that in the case the well-known  $n$ -widths, including the Kolmogorov and entropy  $n$ -widths, do not converge to zero.

Also we consider the problem of learning neural networks from samples. Using a estimates of the pseudo-dimension of the neural networks manifolds and approximation by the manifold, we estimate the sample size which is sufficient for obtaining the almost-optimal stochastic approximation of function classes. These results can be used to solve Smale's network problem.