The entropy in the Learning Theory. Error estimates

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The results of our joint papers with V.N. Temlyakov will be presented. We investigate some problems in learning theory in the setting formulated by F. Cucker and S. Smale. The goal is to find an estimator f_z on the base of given data $z := ((x_1, y_1), \dots, (x_m, y_m))$ that approximates well the regression function f_ρ of an unknown Borel probability measure ρ defined on $Z = X \times Y$. We consider a problem of approximate recovery of a projection f_W of an unknown regression function f_ρ onto a given class of functions W. It is known that the behavior of the entropy numbers $\varepsilon_n(W)$ of W in the uniform norm plays an important role in the above problem. We obtain sharp (in the sense of order) estimates for the error between f_W and f_z for the classes W satisfying $\varepsilon_n(W) \le Dn^{-r}$, $n = 1, 2, \dots$, $|f| \le D$, $f \in W$. We observe that the error estimates exhibit a saturation phenomenon for the range $r > 1/2$. We improve the error estimates by imposing one additional assumption on the relation between f_ρ and W, namely, we assume $f_\rho \in W$. Also, the use of the $L_2(\rho_X)$ norm in measuring the error has motivated us to study the case when we make an assumption on the entropy numbers $\varepsilon_n(W, L_2(\rho_X))$ of W in the $L_2(\rho_X)$ norm.