Influence of the shape of functions on orders of piecewise-polynomial and rational approximation

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Let I be a finite interval, and let $s \in \mathbb{N}_0$. A function $x : I \to \mathbb{R}$ is called s-monotone on I if its divided differences $[x; t_0, \ldots, t_s]$ of order s are nonnegative for all choices of $s+1$ distinct points $t_0, \ldots, t_s \in I$. Note that 0-monotone, 1-monotone, and 2-monotone functions are just non-negative, non-decreasing, and convex functions, respectively. So, we can say that the parameter s characterizes the shape of functions. Denote by Δ_+^s the class of all s-monotone functions on I, and set $\Delta_+^s B_p = \Delta_+^s \cap B_p$ where B_p is the unit ball in the space L_p, $1 \le p \le \infty$.

Let P_n be the space of algebraic polynomials π_n of order $\le n$. Denote by R_n the set of all rational functions $\rho_n = \check{\pi}_n / \hat{\pi}_n$ where $\check{\pi}_n, \hat{\pi}_n \in P_n$. We denote by $\Sigma_{r,n}$ the set of all piecewise polynomials $\sigma_{r,n}$ of order $\le r$ with $\le n$ knots in I. For $s \in \mathbb{N}$ and $1 \le q < p \le \infty$ we obtain exact orders

$$E_n\left(\Delta_+^s B_p, \Sigma_{r,n}\right)_{L_q} \asymp n^{-\min\{r,s\}} \quad \text{and} \quad E_n\left(\Delta_+^s B_p, R_n\right)_{L_q} \asymp n^{-s}$$

of the deviations of the classes $\Delta_+^s B_p$ from the sets $\Sigma_{r,n}$ and R_n, respectively, in L_q.