Solving a functional equation arising in Information theory

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Given a convex function f defined for strictly positive real variables, the so-called Csiszar f-divergence is a function I_f defined for two n-dimensional probability vectors p and q as $I_f(p,q) := \sum_{i=1}^{n} q_i . f(\frac{p_i}{q_i})$. For this generalized measure of entropy to have distance-like properties, especially symmetry, it is necessary and sufficient for f to satisfy the following functional equation : $f(x) = x\, f(\frac{1}{x})$ for all strictly positive x. In the present work (joint with J.E. Martinez-Legaz from Barcelona (Spain)) we determine all the convex solutions of this functional equation by proposing a way of generating all of them. In doing so, existing usual f-divergences are recovered and new ones are proposed.