Geometric aspects of the Lagrange multipliers rule

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In this survey talk we analyse an intrinsic coordinate free meaning of the Lagrange multipliers rule for smooth extremal problems, including problems with nonholonomic constraints. Elementary symplectic geometry provides an adequate language. The analysis results in a far going generalization and clarification of two fundamental realities of Riemannian Geometry: Sectional Curvature and Morse formula for the index of a geodesic.

This talk is dedicated to Vladimir Mikhajlovich Tikhomirov who appreciates so much both sides of mathematical ideas: their universal algorithmic power and their estetic value, and who perhaps complained about the prevalence of the first side in the Lagrange multipliers rule. We hope that this geometric study helps to restore the balance.