Some Problems of Optimal Recovery of Linear Operators

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In the report we will talk about the optimal recovery of functions from their inexactly given spectrum and recovery of solutions of differential equations from inaccurate initial data. In the first case, as an illustration, we consider the optimal recovery of functions from inaccurate information about their Fourier coefficients. Next, for a special one-parameter semi-group of operators, we consider the optimal recovery of the operator at a given value of the parameter from inaccurate information about the values of other parameters. We construct a family of optimal methods. As a consequence we find a family of optimal methods in the problem of optimal recovery for the solution of the heat equation on $\mathbb{R}^d$

$$\frac{\partial u}{\partial t} = \Delta u, \quad u(0, \cdot) = f(\cdot),$$

at the time instant t from their approximate measurements at time instants $t_1 < \ldots < t_n$. We also consider the problem of optimal recovery of the solution for the Dirichlet problem in the half-space

$$\Delta w\,(x, y) = 0, \ (x, y) \in \mathbb{R}^d \times \mathbb{R}, \ y > 0, \ w(\cdot, 0) = f(\cdot),$$

which is to recover the solution on the hyperplane $y = Y$ from its inaccurate measurements on the hyperplanes $y = y_i$, $i = 1, \ldots, n$.

References

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[2] *Magaril-Il'yaev G.G., Osipenko K.Yu.* Optimal recovery of the solution of the heat equation from inaccurate data // Sb. Math. 2009. V. 200. N 5. P. 665-682.